$$\left| = \begin{array}{l}(x^2+xy+xy^2)(x+y+xy)+(x^2y-xy-y^2)(-x^2y-xy^2), \\ (xy+y^2-x^2y)(x+y+xy)+(xy^2+x^2+xy)(-x^2y-xy^2),\end{array}\right.$$

$$\left.\begin{array}{l}(x^2+xy+xy^2)(x^2y+xy^2)+(x^2y-xy-y^2)(x+y+xy) \\ (xy+y^2-x^2y)(x^2y+xy^2)+(xy^2+x^2+xy)(x+y+xy)\end{array}\right|$$

$$=(x^3+2x^2y+2x^2y^2+xy^2+xy^3+x^3y+3x^2y^3+xy^4+x^3y^2-x^4y^2-x^3y^3)^2$$

$$+(x^4y+3x^3y^2+x^3y^3+x^2y^3+x^2y^4+x^3y-x^2y-2xy^2-y^3-xy^3)^2.$$

If $x=1$, $y=2$, as in given problem, I get $5\times13\times61=59^2+22^2$.

If $x=2$, $y=3$, I find $13\times61\times2021=398^2+807^2$ and so on indefinetly.

AVERAGE AND PROBABILITY.

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

14. Proposed by CHARLES E. MYERS, Canton, Ohio.

$\frac{1}{3}$ of all the melons in a patch are not ripe, and $\frac{1}{4}$ of all the melons in the same patch are rotten. the remainder being good. If a man enters the patch on a dark night and takes a melon at random, what is the probability that he will get a good one?

I. Comment by JOHN DOLMAN, Jr., Counsellor-at-Law, Philadelphia, Pennsylvania.

The published solution of Probability problem No. 14 is erroneous. The simplest correct solution is as follows:

If the melon selected is *not unripe* and *not rotten*, it will be good. The chance that it is unripe is $\frac{1}{3}$, therefore the chance that it is not unripe is $\frac{2}{3}$; by similar reasoning, the chance that it is not rotten is $\frac{3}{4}$. Therefore the chance that it is not unripe and not rotten, is $\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{2}$, which is the chance required.

It is not very difficult to point out the error in the published solution. While it is true there cannot be more than $8n$ nor less than $5n$ good melons, it does not follow that $\frac{1}{2}(8n+5n)$ is the average or most probable number, unless it be predicated that all values between $5n$ and $8n$ are equally likely, which is not the case.

Dropping the n's, suppose there are 8 ripe and 4 unripe melons, the three rotten ones may be selected from these 12 in 220 different ways each of equal probability. Now a very simple application of the principles of choice will show that of these ways

4 would leave 8 good melons
48 would leave 7 good melons
112 would leave 6 good melons
56 would leave 5 good melons

giving therefore the numbers 5, 6, 7, and 8 the relative values thus found, and averaging in the ordinary way, we will find that 6 is the average number of good melons, and therefore $\frac{6}{12}$ or $\frac{1}{2}$ is the probability of selecting a good one.

15. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, in New Windsor College, New Windsor, Maryland.

Todhunter proposes: "From a point in the circumference of a circular field a projectile is thrown at random with a given velocity, which is such that the diameter of the field is equal to the greatest range of the projectile: prove the chance of its falling within the field, is $C=2^{-1}-2\pi^{-1}(\sqrt{2}-1),=.236+$." Is this result perfectly correct as to fact?

Comment on the Solution of Problem 15, by JOHN DOLMAN Jr., Philadelphia, Pennsylvania.

I should not presume to criticise the work of so able and celebrated a mathematician as Professor Matz, did I not consider his solutions of Probability No. 15 vicious in their effects upon the minds of students, striking as they do at the root, not only of the doctrine of mean value and probability, but of the integral calculus itself. "Since the projectiles are *thrown* at random they should *fall* at random," is on a par with, "as an arc varies uniformly the sine varies uniformly," or "because acceleration is constant velocity is constant." The third solution is not clear, but how far any given range the favorable chances can be represented by an area, when the projectiles must fall on the arc of a circle, is difficult to understand. Also in the fifth solution it is stated," for any range PD', the projectiles falling on the circular arc DMD' are within the field" and then the angle PAD' is adopted as uniformly varying, without giving any reason for it. However interesting this may be as mathematical legerdemain, its effects are vicious when it is published without proper explanation for in my humble opinion it is more important that your readers learn to reason correctly than that they be taught to integrate ingeniously.

NOTE.—The solution of problem 14 was published without comment for the reason that we considered the solution to be correct, and we confess that we do not yet see the force of Mr. Dolman's argument, though we have not had time to give it much thought.

As to the solutions of problem 15, we hold that the first solution is the only correct solution as that one and that one alone involves the strict literal statement of the problem. It is evident that the number of ways the projectile can be thrown is equal to the surface of a hemisphere whose radius is R. If now we find the surface of that part of this hemisphere any point at which if a projectile be thrown the projectile will fall upon the circular field, diameter R, and then divide this surface by the surface of the hemisphere, the result will be the probability required. This method of solution would have to be accepted by the most critical mind. Professor Matz's first solution involves this principle.—EDITOR.

19. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Find the average area of the circle which is the locus of the middle points of all chords passing through a point taken at random in the surface of a given circle.

I. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.